Introduction

The removal of building deflections consists of uneven raising of buildings with piston hydraulic jacks (Gromysz, 2017). The jack piston extension is limited to 200 mm. If the height of raising a building exceeds this value, it is necessary to periodically underlay the jacks with elements increasing the lifting range. This way, a temporary support is created, consisting of a hydraulic jack, a stack of parallelepiped elements located under the jack and frequently also additional plates (Fig. 1).

The mechanical properties of the stack of elements situated under the jack dictate the properties of the temporary support. In particular, support stiffness depends to the greatest extent on the stack stiffness.

Subject to investigations up till now have been only the parameters of a stack of wooden elements used for elimination of deflections of low structures, with up...
to three stories (Gromysz, 2018). Moreover, the analyses of axially and eccentrically loaded elements have been carried out mainly in relation to reinforced concrete elements (Kabir & Shafei, 2012), steel columns (Nie, Kang, Shen & Yang, 2017) and masonry structures (Bernat-Maso, Gil & Roca, 2015; Moradabadi, Laefer, Clarke & Lourenço, 2015). Respectively, the results of the experimental research concerns mainly glass fibre reinforced polymer-reinforced geopolymer concrete – GFRP-RGC (Elchalakani, Karrech, Dong, Ali & Yang, 2018), steel (Nie et al., 2017), masonry structures (Cavaleri, Failla, Mendola & Papia, 2005; Bernat-Maso, Gil & Escrig, 2015) and composite multi-drum columns (Buzov, Radnić, Grgić & Baloević, 2018).

This paper presents the results of tests of a stack of parallelepiped steel elements used for the removal of deflections of high buildings.

**Temporary support**

The subject of the analysis is a stack of unconnected cuboidal elements with the total length of $l_{st} = 1.0$ m, forming part of the support (Fig. 2). A single element of the stack consists of rolled profiles joined by welding (Fig. 2). The entire support together with a jack will be subject to experimental tests to reflect the actual working conditions of the stack.

In real conditions, the support rests upon the prepared substrate, so the base of support is not displaced or rotated. A piston of a hydraulic jack is ended with a swivel saddle. It can thus be concluded that a static scheme of the support is represented by a one-end-fixed column, loaded with the force $Q_z$ (Fig. 2).

The stiffness of the support $k_{sup,z}$ in the vertical direction, understood as the value of the force $Q_z$ divided by the change $Δl_{sup}$ of its length, results from the serial connection of the jack with the longitudinal stiffness $k_{hj,z}$ and stack $k_{st,z}$. As presented above, a stack of steel elements will be the subject of the analysis only.

Stack stiffness results from the serial connection of $n$ single parallelepiped elements with stiffness $k_{cub,z}$ each. In order to determine the theoretical stiffness $k_{st,z,teor,1m}$ of the stack with the length $l_{st} = 1.0$ m, the stiffness $k_{cub,z}$ of one element should hence be first calculated.

![FIGURE 2. Static scheme of the support and parallelepiped element](image)
and then \( n \) elements should be connected serially. The stiffness of a single parallelepiped element, due to its cross section changing in a stepwise manner, is determined with consideration of two stiffnesses, i.e. \( k_{\text{cub},1} \) and \( k_{\text{cub},2} \) (Fig. 2)

\[
k_{\text{cub},i} = \frac{h_i b_i E}{l_i}
\]

where:
\( h_i, b_i, l_i \) – dimensions presented in Figure 2;
\( E \) – Young’s modulus of S235 steel (210 GPa).

Hence the stiffness of one parallelepiped element determined from the dependence (2)

\[
k_{\text{cub},z,\text{teor}} = \frac{k_{\text{cub},1} k_{\text{cub},2}}{k_{\text{cub},1} + k_{\text{cub},2}}
\]

is 97,800.2 kN·mm\(^{-1}\).

In turn, the theoretical stiffness \( k_{\text{st},z,\text{teor},1m} \) of the stack \( n = 13 \) of elements forming a stack with the length \( l_{\text{st}} = 1 \) m, resulting from the serial connection of elements, determined from the formula (3)

\[
k_{\text{st},z,\text{teor},1m} = \frac{k_{\text{cub},z,\text{teor}}}{n}
\]

equals 7,523.1 kN·mm\(^{-1}\).

**Examination of stack stiffness**

As mentioned above, the aim of the conducted tests is to determine the stiffness \( k_{\text{st},z,1m} \) of a stack with the length \( l_{\text{st}} = 1 \) m, consisting of 13 parallelepiped elements. This stiffness was defined as the change in the force \( Q_z \) value divided by the corresponding change of the stack length:

\[
k_{\text{st},z,1m} = \frac{Q_z}{\Delta l_{\text{st}}}
\]

The observations made when removing the deflections of objects show, that stack stiffness depends on the value of the load \( Q_z \) and on the load phase. Moreover, due to the fact that a stack of unconnected elements is being loaded, stiffness under the increasing load is different than under the lowering load. A program of tests was therefore created in which supports are loaded with the increasing and decreasing force \( Q_z \).

**Test stand**

It is impossible to reproduce in laboratory conditions the conditions similar to real conditions, where a stack would be loaded with gravity force with the value of up to 1,000 kN, while ensuring the freedom of stack deformations and jack head displacements in all three directions. It was therefore necessary to apply a substitute scheme. The actual support, wherein the static scheme is a one-end-fixed column with the length \( l_{\text{sup}} \), was replaced by a system consisting of two supports directed to each other with bases (Fig. 3). A simply supported system with the length \( 2l_{\text{sup}} \) was therefore examined. The character of the actual support’s behaviour, shown in Figure 3, was maintained here without the need to ensure the free displacement of the loaded end of the element. It was only necessary to ensure its rotation.
The system mentioned was installed in a horizontal position on a test stand (Fig. 4). Support was provided under each of the elements in the form of bearings moving along a polished steel plate. Movement resistance was minimized this way and the freedom of deformations along the axis z, of displacements along the axis x and of rotations along the axis y are ensured for the entire examined system.

A measuring system consisting of a force transducer, LVDT sensors, computer with software, designed for data acquisition, and an analogue-to-digital converter, was applied for measuring displacements and forces. The measurement accuracy of forces and displacements was 1%.

**Programme and progress of tests**

A system with the length of \(2l_{st} = 2.0\) m (Fig. 4), loaded with the cyclic force \(Q_z\), was tested. The load was changed from zero to the maximum value \(Q_{z,\text{max}}\).
Seven \( Q_{z,\text{max}} \) values were adopted, equal to: 50, 75, 100, 250, 500, 750, 1,000 kN. Seven tests of the system were altogether carried out, with four load – unload cycles in each test. Twenty seven hysteresis loops were obtained this way. The force \( Q_z \) was changing during the test with the rate of about 20 kN·s\(^{-1}\), which corresponds to actual conditions during building’s rectification. The designations of successive tests together with successive values of the force \( Q_z \) are shown in Table 1. The longitudinal force \( Q_z \) and variations in the length of the sections \( 2\Delta l_{st,1} \) and \( 2\Delta l_{st,2} \) (Fig. 5) were measured during the tests.

### TABLE 1. Designations of tests and range of loads implemented in individual tests

<table>
<thead>
<tr>
<th>Test designation</th>
<th>Successive values of the force – ( Q_z ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>st/2000/0/0-50/comp</td>
<td>0→50→0→50→0→50→0→50→0→50→0→50→0→50→0→50→0</td>
</tr>
<tr>
<td>st/2000/0/0-75/comp</td>
<td>0→75→0→75→0→75→0→75→0→75→0→75→0→75→0→75→0</td>
</tr>
<tr>
<td>st/2000/0/0-100/comp</td>
<td>0→100→0→100→0→100→0→100→0→100→0→100→0→100→0</td>
</tr>
<tr>
<td>st/2000/0/0-250/comp</td>
<td>0→250→0→250→0→250→0→250→0→250→0→250→0→250→0</td>
</tr>
<tr>
<td>st/2000/0/0-500/comp</td>
<td>0→500→0→500→0→500→0→500→0→500→0→500→0→500→0</td>
</tr>
<tr>
<td>st/2000/0/0-750/comp</td>
<td>0→750→0→750→0→750→0→750→0→750→0→750→0→750→0</td>
</tr>
<tr>
<td>st/2000/0/0-1000/comp</td>
<td>0→1000→0→1000→0→1000→0→1000→0→1000→0→1000→0</td>
</tr>
</tbody>
</table>

Figure 6 show four full load-unload cycles obtained as a result of the carried out tests st/2000/0/0-50/comp (a test corresponding to the smallest value of the force \( Q_{z,\text{max}} = 50 \) kN) and st/2000/0/0-1000/comp (a test corresponding to the highest value of the force \( Q_{z,\text{max}} = 1,000 \) kN). The analogous cycles were obtained for all the tests corresponding to the values \( Q_{z,\text{max}} \).

The hysteresis loops \( Q_z – \varepsilon_{st,z} \), corresponding to all four cycles in a given test, are coinciding, so one of them was only presented below. The superimposed

**Tests results and their analysis**

**Course of the dependence \( Q_z – \varepsilon_{st,z} \)**

The length change of the examined stack with the original length \( 2l_{st} \) is calculated based on the measured changes in the length of sections \( 2\Delta l_{st,1} \) and \( 2\Delta l_{st,2} \) according to the formula

\[
2\Delta l_{st} = \frac{2 \Delta l_{st,1} + 2 \Delta l_{st,2}}{2}
\]

Stack deformation \( \varepsilon_{st,z} \) was defined as

\[
\varepsilon_{st,z} = \frac{2\Delta l_{st}}{2l_{st}}
\]
The courses of loops from all the tests are presented in Figure 7.

The courses of graphs of the function $Q_z - \varepsilon_{st,z}$ obtained in all the tests are coinciding in the common ranges of the load. The phases (Fig. 7) can be distinguished in all the tests on the charts $Q_z - \varepsilon_{st,z}$. The first phase corresponds to the monotonic increase in the load from zero to $Q_{z,\text{max}}$. The dependence $Q_z - \varepsilon_{st,z}$ in this phase is non-linear because there is a clear increase in the slope of the curve along with the rising load. During unloading, the linear dependence $Q_z - \varepsilon_{st,z}$ is first observed. Then, along with a further decrease in the load, the non-linear dependence $Q_z - \varepsilon_{st,z}$ is observed because along with the falling $Q_z$, the slope of the curve $Q_z - \varepsilon_{st,z}$ decreases. Based on such observations, in order to conduct a further analysis, it was decided to divide the hysteresis loop $Q_z - \varepsilon_{st,z}$ to three phases, as shown in Figure 8. The following was therefore distinguished.

Phase I of the load with the non-linear dependence $\varepsilon_{st,z} - Q_z$. The load is rising here from zero to $Q_{z,\text{max}}$, and the deformation to the value $\varepsilon_{st,z,\text{max}}$.

Phase II of unloading with the linear dependence $\varepsilon_{st,z} - Q_z$. The load value decreases to $Q_{z,\text{max}}$ from the agreed limit.
The value $Q_{z,UL}$ was read from charts and was between 0.70 to 0.82 of the value $Q_{z,max}$. The strain changed from $\varepsilon_{st,z,max}$ to the agreed limit of $\varepsilon_{st,z,UL}$.

Phase III including the unloading process, in which the graph $\varepsilon_{st,z} - Q_z$ has a non-linear course. The load value decreases from $Q_{z,UL}$ to zero, and deformation from $\varepsilon_{st,z,UL}$ to zero.

The values of forces $Q_z$ and of deformations $\varepsilon_{st,z}$, being the limits of particular phases, are presented in Table 2.

**TABLE 2. Limits of phases**

<table>
<thead>
<tr>
<th>Test designation</th>
<th>$Q_{z,max}$ [kN]</th>
<th>$\varepsilon_{st,z,max}$ [%]</th>
<th>$Q_{z,UL}$ [kN]</th>
<th>$\varepsilon_{st,z,UL}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>st/2000/0/0-50/comp</td>
<td>52.92</td>
<td>1.485</td>
<td>39.20</td>
<td>1.419</td>
</tr>
<tr>
<td>st/2000/0/0-75/comp</td>
<td>79.50</td>
<td>1.796</td>
<td>65.63</td>
<td>1.774</td>
</tr>
<tr>
<td>st/2000/0/0-100/comp</td>
<td>98.33</td>
<td>1.969</td>
<td>79.33</td>
<td>1.928</td>
</tr>
<tr>
<td>st/2000/0/0-250/comp</td>
<td>249.78</td>
<td>2.872</td>
<td>204.39</td>
<td>2.832</td>
</tr>
<tr>
<td>st/2000/0/0-500/comp</td>
<td>507.12</td>
<td>3.625</td>
<td>413.76</td>
<td>3.590</td>
</tr>
<tr>
<td>st/2000/0/0-750/comp</td>
<td>757.03</td>
<td>4.035</td>
<td>599.70</td>
<td>3.993</td>
</tr>
<tr>
<td>st/2000/0/0-1000/comp</td>
<td>1 008.27</td>
<td>4.315</td>
<td>807.57</td>
<td>4.273</td>
</tr>
</tbody>
</table>

The courses of the dependence $Q_z - \varepsilon_{st,z}$ of successive phases can be approximated with successive approximation functions $f_i$. Phase I was approximated with the function $f_I$ in the form (7), phase II was approximated with the function $f_{II}$ in the form (8) and Phase III was approximated with the function $f_{III}$ in the form (9).

\[
f_I : Q_z(\varepsilon_{st,z}) = \alpha_I e^{\beta_I \varepsilon_{st,z}}, \quad (7)
\]

\[
f_{II} : Q_z(\varepsilon_{st,z}) = \alpha_{II} \varepsilon_{st,z} + \beta_{II}, \quad (8)
\]

\[
f_{III} : Q_z(\varepsilon_{st,z}) = \alpha_{III} e^{\beta_{III} \varepsilon_{st,z}}, \quad (9)
\]

The coefficients $\alpha_I$, $\alpha_{III}$ and $\beta_I$, $\beta_{III}$ were determined with the least squares method. In turn, the coefficients $\alpha_{II}$ and $\beta_{II}$ were determined by creating an equation for a straight line coming through two points which constituted the limit of phases I and III.

Figure 9 shows the examples of graphs of the function $f_I$ superimposed on the results of experimental tests:
st/2000/0/0-50/comp (a test corresponding to the smallest value of the force \(Q_{z,\text{max}} = 50 \text{kN}\)) and st/2000/0/0-1000/comp (a test corresponding to the highest value of the force \(Q_{z,\text{max}} = 1,000 \text{kN}\)). The analogous graphs can be of course created for all the tests. Satisfactory consistency of tests results with the description of the dependencies (7), (8), (9) is seen here.

**Stack stiffness** \(k_{st,z}\)

As a result of deriving the functions \(f_i\) approximating the courses of the dependencies \(Q_z - \varepsilon_{st,z}\), the dependencies (10) are obtained describing the stiffnesses \(k_{st,z,i}\) as the functions of strains.

\[
k_{st,z,i}(\varepsilon_{st,z}) = \frac{df_i}{d\varepsilon_{st,z}} \quad (10)
\]

Hence the functions of stiffness in particular phases have the forms

\[
k_{st,z,\text{I}}(\varepsilon_{st,z}) = \alpha_1 \beta_1 e^{\beta_1 \varepsilon_{st,z}} \quad (11)
\]

\[
k_{st,z,\text{II}}(\varepsilon_{st,z}) = \alpha_\text{II} \quad (12)
\]

\[
k_{st,z,\text{III}}(\varepsilon_{st,z}) = \alpha_\text{III} \beta_\text{III} e^{\beta_\text{III} \varepsilon_{st,z}} \quad (13)
\]

The reverse functions \(f_i^{-1}\) and \(f_{\text{III}}^{-1}\) were determined to examine the effect of the load value on stack stiffness.

\[
f_i^{-1}(Q_z) = \frac{\ln\left(\frac{Q_z}{\alpha_i}\right)}{\beta_i}, \quad i = \text{I, III} \quad (14)
\]

The dependence (14) describes the strain \(Q_{st,z}\) as the function of the load \(Q_z\). The following dependence (15) is obtained after substituting (14) to the equations (11) and (13):

\[
k_{st,z,i}(Q_z) = \alpha_i \beta_i e^{\beta_i \varepsilon_{st,z}}, \quad i = \text{I, III} \quad (15)
\]

which, after transformation, assumes the form

\[
k_{st,z,i}(Q_z) = B_i Q_z, \quad i = \text{I, III} \quad (16)
\]

It turns out that the stiffnesses \(k_{st,z,\text{I}}\) and \(k_{st,z,\text{III}}\) of the stack are the linear functions of the load \(Q_z\). The stiffness is therefore proportional to the load and increases along with the increasing load force. At the same time, the directional coefficients of the dependence (16) are equal to the coefficients \(\beta_1\) and \(\beta_\text{III}\) – determined with the least squares method – of, respectively, functions \(f_1\) and \(f_{\text{III}}\).

The derived function \(k_{st,z,\text{II}}\) has the constant value \(\alpha_\text{II}\) in phase II and seems to be independent from the variable \(Q_z\). This constant assumes however different values depending on the value of the maximum force \(Q_{z,\text{max}}\). It turns out that in the analysed load range of \(Q_{z,\text{max}}\), stack stiffness in phase II varies approximately linearly in accordance with equation (17).

\[
k_{st,z,\text{II}}(Q_z) = 4,487 Q_z, \quad (Q_z) \leq Q_{z,\text{max}} \quad (17)
\]

where:

\(Q_{z,\text{max}}\) – maximum load \([\text{kN}]\);

\(k_{st,z,\text{I}}\) – stiffness in phase II \([\text{kN} \cdot \text{‰}^{-1}]\).

Figures from 10 to 12 show graphs of the stiffness \(k_{st,z,\text{I}}\) from the load \(Q_z\) over the successive phases.
The maximum stiffnesses $k_{st,z,\text{max},I}$ for all the phases were achieved for the highest load with the force $Q_z$.

After conducting the tests and describing them it is possible to compare the values of stack stiffness determined theoretically with the dependencies prepared on the basis of the tests. The theoretical analysis shows that stiffness $k_{st,z,\text{teor}}$ of the stack with the length of $l_{st} = 1\,\text{m}$, consisting of $n = 13$ elements is constant and equals $7,523.1\,\text{kN} \cdot \text{mm}^{-1}$ (the value determined on the basis of eq. 3). It was revealed by analysing the results of tests however that the stiffness is not a constant value and depends on the value of the load $Q_z$. Table 3 lists maximum stiffnesses corresponding to...
three phases for the load $Q_{\text{max}} = 1,000$ kN, determined on the basis of the dependence (15), (16) and (17). The values were then compared, calculated according to equations (16) and (17), with the one determined according to equation (3), by introducing the parameter

$$\eta_{k,\text{st},z} = \frac{k_{\text{st},z,i}}{k_{\text{st},z,\text{teor}}} \cdot 100\%$$  \hspace{1cm} (18)

The values of this parameter are provided in the last column of Table 3.

It is seen by analysing the values provided in the Table 3 that the stiffnesses determined in experimental tests are even ten times smaller than the stiffnesses determined by calculations. This difference results from the deformations that occur at the interface between parallelepiped elements and deformations of the elements forming the stack. The deformations at the interface of elements result from the fact that the contact surface is not a perfect plane. In turn, higher deformations of the parallelepiped elements themselves result probably from the susceptibility of welding joints connecting the rolled parts, from which the parallelepiped elements were made.

### TABLE 3. Comparison of stiffnesses

<table>
<thead>
<tr>
<th>Phase</th>
<th>$Q_z$ [kN]</th>
<th>$k_{\text{st},z,\text{max},i}$ [kN·‰⁻¹]</th>
<th>$k_{\text{st},z,\text{teor}}$ [kN·‰⁻¹]</th>
<th>$\eta_{k,\text{st},z}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1 008.3</td>
<td>983.1</td>
<td>7 523.1</td>
<td>13.1</td>
</tr>
<tr>
<td>II</td>
<td>1 008.3–807.6</td>
<td>4 750.1</td>
<td>7 523.1</td>
<td>63.1</td>
</tr>
<tr>
<td>III</td>
<td>807.6</td>
<td>762.2</td>
<td>7 523.1</td>
<td>10.1</td>
</tr>
</tbody>
</table>

### Conclusions

It is critical to determine correctly the stiffness of the stack of steel parallelepiped elements forming part of temporary building supports to correctly model the building deflection removal process. During the rectification, the stack forming part of the temporary support is loaded in a cyclic manner. Tests were designed and carried out for this reason, in which a stack of parallelepiped elements was loaded with the increasing and decreasing axial force $Q_z$.

Three distinct phases of the $Q_z - \varepsilon_{\text{st},z}$ dependence course were identified in each of the conducted load – unload cycle tests. The loading process was described with one phase (phase I), whereas the unloading process with two phases (phases II and III).

The stack stiffness for the examined range of loads was not constant. The stiffness in phases I and III was varying in a linear manner along with the change of the load $Q_z$. In phase II, stiffness is indeed constant, however, its value depends on the value $Q_{z,\text{max}}$, from which unloading takes place. This dependence is also linear.

The highest stack stiffness in all the phases was achieved during the test designated as st/2000/0/0-1000/comp, where $Q_{z,\text{max}}$ reached the value of 1,008.3 kN. Such stiffnesses were, respectively, 13.1, 63.1 and 10.1% of the theoretical stiffness.

A non-linear behaviour of the stack results from its untypical construction. The elements of the stack are not permanently interconnected, and their coopera-
Examination of stiffness of stack of steel elements forming part of supports used for removal of building deflections.

The removal of building deflections consists of uneven raising of buildings with piston hydraulic jacks. A stack of parallelepiped steel elements is situated under jacks during deflection removal for technological reasons. The stack has decisive influence on the stiffness of the supports. Tests of the stack of parallelepiped elements loaded with increasing and decreasing force were designed and carried out. Three characteristic phases were identified based on the tests. The maximum stiffness in particular phases was, respectively 13.1, 63.1 and 10.1% of theoretical stiffness.

Summary

Examination of stiffness of stack of steel elements forming part of supports used for removal of building deflections.

The removal of building deflections consists of uneven raising of buildings with piston hydraulic jacks. A stack of parallelepiped steel elements is situated under jacks during deflection removal for technological reasons. The stack has decisive influence on the stiffness of the supports. Tests of the stack of parallelepiped elements loaded with increasing and decreasing force were designed and carried out. Three characteristic phases were identified based on the tests. The maximum stiffness in particular phases was, respectively 13.1, 63.1 and 10.1% of theoretical stiffness.

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