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Determination of stress-strain state of reinforced slabs from non-linear material taking into account the influence of aggressive environment*

Key words: finite elements, reinforced plates, chloride-containing operating environments, polymer, non-linear sensitivity to the type of tense state

Introduction

The materials considered in this work – reinforced concrete and polymer-concrete, as structural materials have been widely distributed in the construction of buildings of all kinds and purpose. However, different resistance inherent in concretes (Treschev, 2016; Seljaev, Punchina, Tereshkin, Seljaev & Kechutkina, 2018) does not allow using classical mechanics of solids methods in strength calculations of constructions. It should also be noted that some types of concrete have a significant non-linearity of the deformation diagrams (Treschev, 2016; Seljaev et al. 2018) when changing the kind of tense state.

In addition, the structures are often exposed to aggressive operating environments, resulting in a change in physical and mechanical properties of the material. One of the most widespread aggressive operating environments is sodium chloride (NaCl), which has both natural character (salty seawater and sea fog), and techno genic (everywhere is found in the composition of anti-ices and various technological environments).

*Due to complexity of material article was formatted in one-column page style.

The analysis of known determinants for materials with complicated properties, exploited in aggressive environments, has shown that existing theories and methods have a number of disadvantages, not taking into account the important peculiarities of material deformation, which in many cases leads to significant errors of the obtained approximations of experimental data.

The most universal and consistent theory of deformation of different resistant materials is the model of Treschev proposed by the author (2016) and the most adequate equations describing the changes in the mechanical properties of materials under the influence of an aggressive environment are presented by Seljaev et al. (2018). In their theory, authors propose to use normalized stresses for the description of properties of different resistant structurally isotropic and anisotropic materials. Numerical analysis of the theory was carried out in various works, among which a study conducted by Parfenov and Okusok (2017) in calculating the stress-strain state of reinforced concrete plates, passed a deep experimental study in the experiments of Bach and Graf (1915) as well as Gehler and Amos (1935), recognized by Karpenko (1976).

In turn, the method of accounting kinetics of aggressive environment, proposed by Petrov and Penina (2008) Seljaev et al. (2018), allows the most correct and full description the kinetics of aggressive environment.

Thus, the authors of the presented theory, summarizing the two above-mentioned research directions, offer a mathematical model of calculating the stress-strain state of reinforced concrete structures taking into account the influence of external operating environments and different resistant material.

Statement of the problem, basic provisions

It is proposed to solve this problem using the modification of the hybrid FE with five degrees of freedom in the node and the stiffness matrix obtained directly for the arbitrary flat triangular element (Petrov & Penina, 2008). This final element is developed based on two modifications of the hybrid FE proposed by Cook (1972).

Construction of finite-elemental model of definition of stress-strain state of layered reinforced slabs from non-linear material is described in detail by Treschev, Telichko and Bashkatov (2014).

The task of bending the reinforced concrete plates, regardless of the geometrical configuration, is proposed to be considered in the conditions of active deformation and simple loading, while the authors use the potential of deformations presented by Treschev (2016), in the “framework” of which the elastic-plastic properties of concrete are stacked as non-linear material:

$$\begin{aligned} W_1 &= \left(A_e + B_e\xi\right)\sigma^2 + (C_e + D_e\xi + E_e\eta\cos 3\varphi)\tau^2 + \\ &+ [\left(A_p + B_p\xi\right)\sigma^2 + \left(C_p + D_p\xi + E_p\eta\cos 3\varphi\right)\tau^2]^n, \end{aligned} \tag{1}$$

where:
A_e, B_e, C_e, D_e, E_e, A_p, B_p, C_p, D_p, E_p – the constants of potential that are subjects to experimental determination, experiments are taken from the works of Gvozdev and Kasimov;
$\xi = \sigma / S_0$, $\eta = \tau / S_0$ – normalized normal and tangent stresses on the octahedral site;
$S_0 = \sqrt{\sigma^2 + \tau^2}$ – the module of the vector of full voltage on the octahedral site;
$\sigma = \delta_{ij}\sigma_{ij} / 3$ and $\tau = \sqrt{S_{ij}S_{ij} / 3}$ – regular and tangent stresses;
$\cos 3\varphi = \sqrt{2}\det(S_{ij}) / \tau^3$;
φ – the phase of stress; $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma$.

In view of heterogeneity of structure on a thickness, it is necessary to break it down on a number of fictitious layers. In this case, depending on the specific conditions of stress-strain state of fictitious layers, select the following groups: (a) polymer-concrete layer; (b) concrete layers without cracks; (c) reinforced (reinforced concrete layers) without cracks; (d) concrete layers with cracks; (e) reinforced (reinforced concrete layers) with cracks; (f) reinforced (reinforced concrete layers) with overlapping cracks.

For modeling of concrete layers without cracks, differentiating on components of tensor stresses the potential of deformations can be distinguished from the received expressions the matrix of connection of deformations and stresses:

$$\{e\} = [A]\{\sigma\}, \tag{2}$$

where:

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & A_{14} & A_{15} \\ & A_{22} & A_{26} & A_{24} & A_{25} \\ & & A_{66} & A_{64} & A_{65} \\ & \text{sim} & & A_{44} & A_{45} \\ & & & & A_{55} \end{bmatrix}. \tag{3}$$

With this A_{11}, A_{12}, A_{16}, A_{14}, A_{15}, A_{22}, A_{26}, A_{24}, A_{25}, A_{66}, A_{64}, A_{65}, A_{44}, A_{45}, A_{55} – the components of a symmetric matrix $[A]$ are defined through R_i – constants of a potential W_1 (Seljaev et al., 2018),

where:

$$A_{11} = \{2(R_1 + 2R_2)/3 + R_3\xi(1-\xi^2)/3 + R_4[\xi(2-\eta^2) + 4(\sigma_{11} - 2\sigma_{22})/9S_0] +$$
$$+R_5[\eta\cos 3\varphi(1+\xi^2) + 2\sqrt{2}\xi - 2\cos 3\varphi - \sqrt{2}\sigma_{22} / S_0]\}/3;$$

$$A_{12} = \{2(R_1 - R_2)/3 + (R_3 + R_4/3)\xi + R_5[\cos 3\varphi(1-\xi) - \sqrt{2}\xi]\}/3;$$

$$A_{16} = \left(2R_4/3 + \sqrt{2}R_5\right)\tau_{12}/3S_0;$$

$$A_{16} = A_{26};$$

$$A_{14} = \left(2R_4/3 + \sqrt{2}R_5\right)\tau_{13}/3S_0;$$

$$A_{15} = 2\left(R_4/3 - \sqrt{2}R_5\right)\tau_{23}/3S_0;$$

$$A_{22} = \{2(R_1 + 2R_2)/3 + R_3[\xi(1-\xi^2)/3 + R_4[\xi(2-\eta^2) + 4(\sigma_{22} - 2\sigma_{11})/9S_0] +$$
$$+ R_5[\eta\cos 3\varphi(1+\xi^2) + 2\sqrt{2}\xi - 2\cos 3\varphi - \sqrt{2}\sigma_{11}/S_0]\}/3;$$

$$A_{24} = \left(2R_4/3 - \sqrt{2}R_5\right)\tau_{13}/3S_0;$$

$$A_{25} = \left(2R_4/3 + \sqrt{2}R_5\right)\tau_{23}/3S_0;$$

$$A_{66} = 2\{2R_2 - R_3\xi^3 + R_4[\xi(2-\eta^2) - (\sigma_{11} + \sigma_{22})/3S_0] +$$
$$+ R_5[\sqrt{2}(\sigma_{11} - \sigma_{22})/2 - \eta^3\cos 3\varphi]\}/3;$$

$$A_{64} = \sqrt{2}R_5\tau_{23}/S_0; \; A_{65} = \sqrt{2}R_5\tau_{13}/S_0;$$

$$A_{44} = 2\{2R_2 - R_3\eta^3 + R_4[\xi(2-\eta^2) - (\sigma_{11} + \sigma_{22})/3S_0] +$$
$$+ R_5[\sqrt{2}\eta(\sigma_{11} - 2\sigma_{22})/2 - \eta^3\cos 3\varphi]\}/3;$$

$$A_{45} = \sqrt{2}R_5\tau_{12}/S_0;$$

$$A_{55} = 2\{2R_2 - R_3\eta^3 + R_4[\xi(2-\eta^2) - (\sigma_{11} + \sigma_{22})/3S_0] +$$
$$+ R_5[\sqrt{2}\eta(\sigma_{22} - 2\sigma_{11})/2 - \eta^3\cos 3\varphi]\}/3.$$

For each of the fictitious concrete layers of the finite element, the elasticity matrix [B] can be expressed through the flexibility matrix in the form (3):

$$[B] = [A]^{-1}. \tag{4}$$

Having introduced stresses in the concrete layer as the sum of stresses in concrete and armature, we get a matrix of elasticity for the reinforced layers:

$$[B] = [A]^{-1} + [B_S], \tag{5}$$

where $\left[B_S\right]=\begin{bmatrix} B_{S11} & 0 & 0 & 0 & 0 \\ & B_{S22} & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ & sim & & 0 & 0 \\ & & & & 0 \end{bmatrix}$, $B_{S11}, = E_S\mu_{11}$, $B_{S22} = E_S\mu_{22}$, E_S is the modulus of reinforcement material elasticity; $\mu_{11} = A_{Si} / S_{i11}h_S$, $\mu_{22} = A_{Si} / S_{i22}h_S$ – reinforcement ratios in the corresponding directions; A_{Si} – cross-sectional area of the reinforcing bar; S_{i11}, S_{i22} – step of rods parallel to the axes X_1 and X_2; h_S – total thickness of reinforced layers.

For a concrete layer we believe that cracks will be formed if the condition (6) is fulfilled:

$$\sigma_{11}^2+\sigma_{22}^2+3\cdot\left(\tau_{12}^2+\tau_{23}^2+\tau_{13}^2\right)-\left(\sigma_{11}\sigma_{22}\right)-\left(R_{bt}-R_b\right)\left(\sigma_{11}+\sigma_{22}\right)-R_{bt}R_b>0, \tag{6}$$

where:

$\sigma_{11},\sigma_{22},\tau_{12},\tau_{13},\tau_{23}$ – stresses in concrete at the time of cracking, calculated for the center of the fictitious layer;

R_{bt}, R_b – the tensile strength of concrete at axial stretching and compression, respectively.

Let us admit that with the appearance of cracks the concrete layer in the area of this finite element stops working, therefore, the matrix for concrete layers with cracks will take the form:

$$[\mathrm{B}] = 0. \tag{7}$$

As a criterion for starting the cracking for the RC layer, use the condition

$$\sigma_{B11}^2+\sigma_{B22}^2+3\cdot\left(\tau_{12}^2+\tau_{23}^2+\tau_{13}^2\right)-\left(\sigma_{B11}\sigma_{B22}\right)-\left(R_{bt}-R_b\right)\left(\sigma_{B11}+\sigma_{B22}\right)-R_{bt}R_b>0, \tag{8}$$

where σ_{Bij} – the stresses in concrete of an RC layer.

The direction of crack development is proposed to determine the angle χ_1 between the normal to the crack and the axis x_1:

$$\chi_1 = \mathrm{arctg}[(\sigma_{B1t}-\sigma_{B11})/\tau_{12}]. \tag{9}$$

where σ_{B1t} – the greatest of the main stretching stresses in concrete.

For cracked in reinforced concrete layers for directions along cracks, where its integrity is not violated physically non-linear properties of concrete approximate sectional modulus of elasticity E_B and cutting coefficient of transverse deformations v_B, defined from equation:

$$e_{22}^{*} = A_{12}^{*}\sigma_{B11}^{*} + A_{22}^{*}\sigma_{B22}^{*} = \left(\sigma_{B22}^{*} - \nu_B \sigma_{B11}^{*}\right)/E_B, \tag{10}$$

i.e. $E_B = 1/A_{22}^{*}$; $\nu_B = -A_{12}^{*}/A_{22}^{*}$, where A_{12}^{*}, A_{22}^{*} – the components of the flexibility matrix are calculated in an orthogonal coordinate system $X_1^{*}OX_2^{*}$ rotated relative to the source system X_1OX_2 at an angle χ_1.

Then the dependencies between deformations and stresses in the rotated coordinate system are determined by the expression:

$$\left\{e^{*}\right\} = \left[A^{*}\right]\left\{\sigma_B^{*}\right\}, \tag{11}$$

where:

$$\left\{e^{*}\right\} = \begin{Bmatrix} e_{11}^{*} \\ e_{22}^{*} \\ \gamma_{12}^{*} \\ \gamma_{13}^{*} \\ \gamma_{23}^{*} \end{Bmatrix}; \left[A^{*}\right] = \begin{bmatrix} A_{11}^{*} & A_{12}^{*} & 0 & 0 & 0 \\ & A_{22}^{*} & 0 & 0 & 0 \\ & & A_{66}^{*} & 0 & 0 \\ & Sim & & A_{44}^{*} & 0 \\ & & & & A_{55}^{*} \end{bmatrix}; \left\{\sigma_B^{*}\right\} = \begin{Bmatrix} \sigma_{B11}^{*} \\ \sigma_{B22}^{*} \\ \tau_{B12}^{*} \\ \tau_{B13}^{*} \\ \tau_{B23}^{*} \end{Bmatrix};$$

$$A_{11}^{*} = 1/\left(E_B\omega\right);\ A_{12}^{*} = -\nu_B/E_B;\ A_{22}^{*} = 1/E_B;$$
$$A_{44}^{*} = A_{66}^{*} = 2\left(1+\nu_B\right)/\left(E_B\omega\right);\ A_{55}^{*} = 2\left(1+\nu\right)/E_B.$$

where the concrete deformation module is determined by magnitude $E_B\omega$ (ω – a function that determines the degree of destruction of concrete $0 < \omega \le 1$) (Treschev & Telichko, 2003b).

The flexibility matrix for the cracked of the reinforced layer in the original coordinate system has the form (Treschev & Telichko, 2003b):

$$\left[A^{c}\right] = \begin{bmatrix} A_{11}^{c} & A_{12}^{c} & A_{16}^{c} & 0 & 0 \\ & A_{22}^{c} & A_{26}^{c} & 0 & 0 \\ & & A_{66}^{c} & 0 & 0 \\ & \text{sim} & & A_{44}^{c} & 0 \\ & & & & A_{55}^{c} \end{bmatrix}. \tag{12}$$

Accordingly, the elastic matrix for reinforcement cracked reinforced concrete layer in the original coordinate system X_1OX_2 will take the form:

$$\left[B_S^c\right]=\begin{bmatrix} E_{S11}\mu_{11} & 0 & 0 & 0 & 0 \\ & E_{S22}\mu_{22} & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ & \text{sim} & & 0 & 0 \\ & & & & 0 \end{bmatrix}, \tag{13}$$

where E_{S11}, E_{S22} – the cutting modules of the reinforcement material deformation, respectively, along the axes X_1 and X_2 that are determined from the condition $E_{skk}=\begin{cases} E_s \text{ при } \sigma_{skk}<\sigma_p\mu_{kk} \\ \sigma_p/e_{kk} \text{ при } \sigma_{skk}\geq\sigma_p\mu_{kk} \end{cases}$; σ_p – yield stress of reinforcement material; $k = 1, 2$.

The matrix of elasticity of the concrete layer will take the form:

$$\begin{gathered}[B]=\left[B_B^c\right]+\left[B_S^c\right],\\ \left[B_B^c\right]=\left[A^c\right]^{-1}.\end{gathered} \tag{14}$$

It is necessary to define the function of damage ω, which is calculated through a coefficient ψ_S that takes into account the work of stretched concrete on the areas located between the cracks (Treschev & Telichko, 2003b):

$$\psi_S=E_{sn}/\left(E_B\omega+E_{sn}\right), \tag{15}$$

where E_{sn} is the reinforcement modulus in the direction along the normal to the crack,

$$E_{sn}=E_{S11}\mu_{11}\cos^4\chi_1+E_{S22}\mu_{22}\sin^4\chi_1 \tag{16}$$

from which the function ω:

$$\omega=\left(E_{S11}\mu_{11}\cos^4\chi_1+E_{S22}\mu_{22}\sin^4\chi_1\right)\left(1/\psi_s-1\right)/E_B. \tag{17}$$

Using the method of successive approximations the matrix $[A^C]$ and components of the elasticity matrix $[B]$ are determined by the calculated function ω and coefficient ψ_S.

If the stresses σ^*_{B11} coincide with the value 0.7 R_{bt} (within the accepted error δ) the solution process stops, and the parameters ψ_S, ω and the matrix $[A^C]$ are considered to be finally calculated. Then the components of the elasticity matrix (14) are calculated.

It is considered that in the case of intersecting cracks in RC layer within this finite element the concrete does not work, i.e. the elasticity matrix takes the form

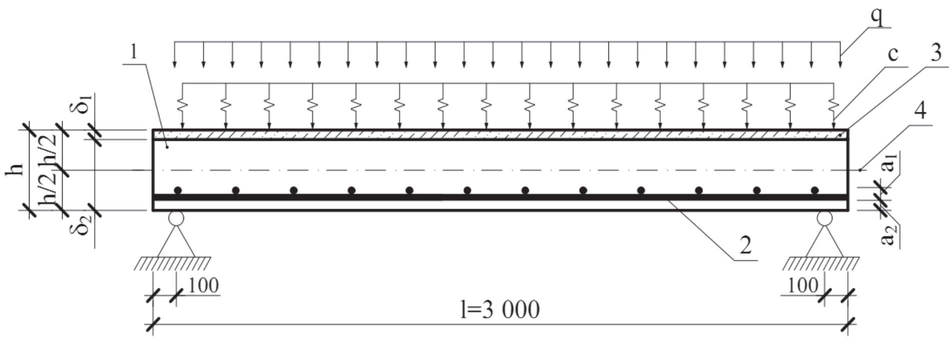
$$[B]=[B_S^c], \tag{18}$$

where the matrix $[B_S^c]$ is determined according to the condition (13).

Additional input models and assumptions, as well as the complete order of construction of fictitious layers in the calculation of such structures are quite fully presented in the works of Treschev and Telichko (2003a).

Simulation of aggressive working environment behavior

As part of this task, the case where the aggressive operating environment is in contact with only the polymer concrete layer located in the compressed area of the slab are analyzed. As a reinforced concrete plate 711, detailed in the experiments of Geler and Amos (1932), the calculation diagram of the task in question is shown in Figure 1.



1 – reinforced concrete layer of the slab; 2 – reinforcement of the slab; 3 – polymer-concrete layer; 4 – median surface; q – uniformly distributed load; c – aggressive environment; δ_1 – thickness of the polymer concrete layer; δ_2 – thickness of the concrete layer; h – full thickness of the slab; a_1 – thickness of the reinforced layer; a_2 – thickness of the protective layer of concrete for the reinforcing mesh.

FIGURE 1. Initial scheme of problem

Experimental studies show that in the process of operation in the material of structures under the influence of the working environment there is a heterogeneity of physical and mechanical properties. Heterogeneous distribution of properties on the section of the structure and kinetics of the development of this process are determined by complex physical and mechanical processes, and depend on the level and nature of the tense state of the material, this type of heterogeneity is called induced heterogeneity (Petrov, Inozemcev & Sineva, 1996; Seljaev, 2006).

Induced heterogeneity is characterized by dependence on the coordinates and concentration of the aggressive medium at an arbitrary point of the material section. We believe that the development of induced heterogeneity can be taken into account by the introduction, along with the criterion of objective strength of the concept of objective diagrams of deformation for fixed moments of time related to the indicator of objective strength and curve long-lasting strength obtained by testing samples in an aggressive environment.

Take as an expression to determine the depth of the layer affected by the medium, the non-linear law proposed by Petrov et al. (1996):

$$\Delta(t) = \alpha\sqrt{t}, \tag{19}$$

where:
$\Delta(t)$ – depth of the layer affected by the medium;
t – the time of influence of an aggressive medium;
α – experimental coefficient, which depends on a particular material-environment pair.

The coefficient α describes the chemical activity of the working environment and the force resistance of the structural material, and in accordance with the experimental data under consideration α = 13.05 mm·year$^{-0.5}$.

In accordance with the theory of Ilyushin and taking into account experiments of Petrov and Selyaev we take the Poisson's ratio to be ν_b = 0.5.

Taking into account the degradation of material properties, the expression of variable section E_c and tangent modules E_k of concrete are accepted in the form proposed by Petrov and Penina (2008) in their studies:

$$\begin{aligned} E_c &= E_c^0 F\left[B(x_3)\right]; \\ E_k &= E_k^0 F\left[B(x_3)\right], \end{aligned} \tag{20}$$

where:
E_c^0 – the sectional module of the material without influence of aggressive operating environment;
E_k^0 – a tangent module without influence of an aggressive environment;
$F(B)$ – function of degradation of a section and tangent modules;
x_3 – a coordinate in the direction of structure thickness.

The results of experimental studies of composite concrete (Seljaev, 2006; Shamshina, 2017a, b; Fedosov, Rumyantseva & Konovalova, 2018; Petrov, Mischenko & Pimenov, 2018) allowed writing down the function of degradation in the form of:

$$\omega_{pb} = F\left[B(x_3)\right] = \exp\left[-\lambda B(x_3)\right] \tag{21}$$

where λ is the relative speed of changing the section and tangent modules.

$$-\lambda = \frac{F'(B)}{F(B)}, \tag{22}$$

where $F'(B)$ – the rate of degradation, differentiation is carried out according to the time parameter.

Note that the impact of the aggressive environment on the polymer layers is not a violation of the acceptability of potential determinant relationships, oriented to nonlinear dilatant and different resistant isotropic material. Accordingly the reasoning given by Petrov et al. (1996), Seljaev (2006) and Petrov and Penina (2008) are fair also for the considered case. Physically nonlinear properties of concrete will be approximate by the sectional modulus of elasticity (E_c) and by the cutting coefficient of transverse deformations (v_c), defined from the following equation:

$$e_{22} = A_{12}\sigma_{11} + A_{22}\sigma_{22} = \left(\sigma_{22} - v_c\sigma_{11}\right)/E_c, \tag{23}$$

i.e. $E_c = 1/A_{22};\ v_c = -A_{12}/A_{22},$

where A_{12}, A_{22} – the components of the flexibility matrix, calculated according to the formulas for the concrete layer without cracks.

Taking into account the stated, dependence between deformations and tensions for the polymer concrete layer by analogy with the modelling of layers of reinforced concrete slab (Trechev & Telichko, 2003), we shall present in the form:

$$\left\{ e^* \right\} = \left[A^* \right] \left\{ \sigma^* \right\}, \tag{24}$$

where

$$\left\{e^*\right\} = \begin{Bmatrix} e_{11}^* \\ e_{22}^* \\ \gamma_{12}^* \\ \gamma_{13}^* \\ \gamma_{23}^* \end{Bmatrix}; \left\{\sigma^*\right\} = \begin{Bmatrix} \sigma_{11}^* \\ \sigma_{22}^* \\ \tau_{12}^* \\ \tau_{13}^* \\ \tau_{23}^* \end{Bmatrix}; \tag{25}$$

$$
\left[A^*\right]=\begin{bmatrix} A_{11}^* & A_{12}^* & 0 & 0 & 0 \\ & A_{22}^* & 0 & 0 & 0 \\ & & A_{66}^* & 0 & 0 \\ & \text{sim} & & A_{44}^* & 0 \\ & & & & A_{55}^* \end{bmatrix}; \tag{26}
$$

$$
\begin{aligned} &A_{11}^* = 1/\left(E_c\omega_{pb}\right);\ A_{12}^* = -\nu_c/\left(E_c\omega_{pb}\right);\ A_{22}^* = 1/\left(E_c\omega_{pb}\right); \\ &A_{44}^* = A_{66}^* = 2\left(1+\nu_c\right)/\left(E_c\omega_{pb}\right);\ A_{55}^* = 2\left(1+\nu\right)/\left(E_c\omega_{pb}\right). \end{aligned} \tag{27}
$$

In this case, the concrete deformation module is defined by the value $E_c\omega_{pb}$ (ω_{pb} – degradation function (Petrov et al., 1996) $0 < \omega_{pb} \le 1$).

As a result, for the polymer-concrete layer have:

$$
[B] = \left[A^*\right]^{-1}. \tag{28}
$$

Sample problem and results

The following characteristics of the slab were used in the calculation: (a) the modulus of elasticity of reinforcing steel was accepted equal to $E_s = 2{\cdot}10^5$ MPa; yield strength of reinforcement is 320 MPa; (b) the polymer concrete layer is adopted from epoxy concrete, whose modulus of elasticity is E_b = 25,500 MPa; (c) the thickness of the polymer-concrete layer – 0.04 m; (d) aggressive environment – 20% solution NaCl, with density r = 1.219 g·cm^{-3}. Dimensions of the slab: 3 × 1.5 × 0.189 m. The time of the environment was considered at intervals from 0 to 30 months, and the load varied from 0 to 50 kPa. The characteristics of the RC part of the slab are described in detail in the papers of Bach and Graf (1915) and Treschev and Telichko (2003a) (referred as 711). The results of the calculation are shown in Figures 2–4. The experimental data extracted from the publications of Petrov and his students performed in the Saratov State University (Saratov) (Petrov et al., 1996; Seljaev, 2006; Fedosov et al., 2018).

The above-mentiored graphs show the presence of quantitative effects associated with consideration of multimodulus behavior of materials, the degradation of a layer of protective material under the influence of an aggressive operating environment and the damage to the carrier layers in the form of a cracking. It is shown that as the concentration of the aggressive medium in the polymer-concrete layer increases, redistribution of stresses and deformations occurs. The increase in deflections of the plate before the formation of cracks reaches 17% and after the formation of cracks

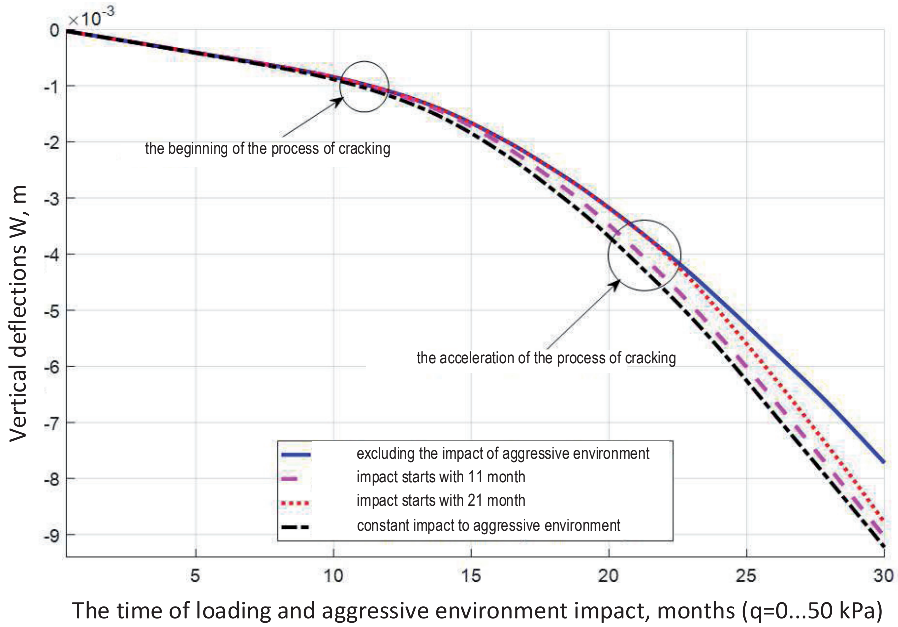


FIGURE 2. Dependence of vertical deflections from the beginning and the period of operation of the aggressive operating environment

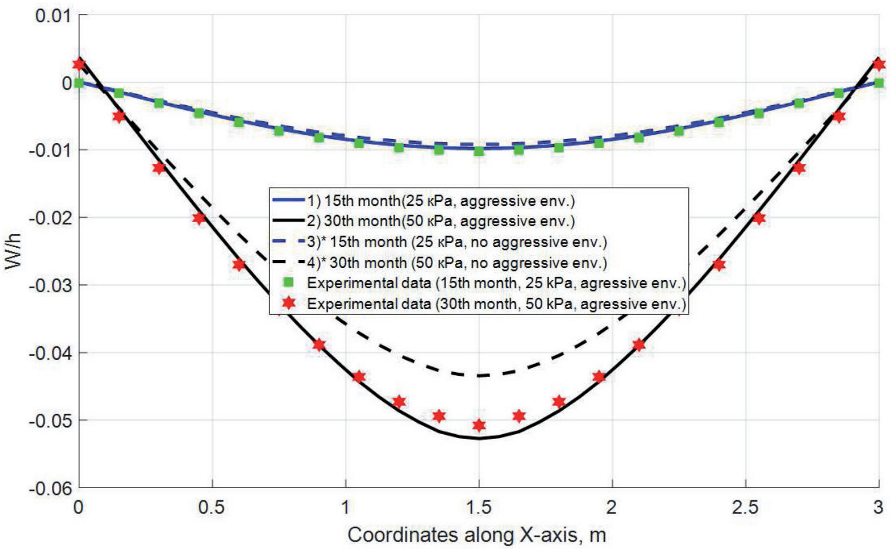


FIGURE 3. Middle plane deflections along the OX axis (the axis directed along the long side of the slab)

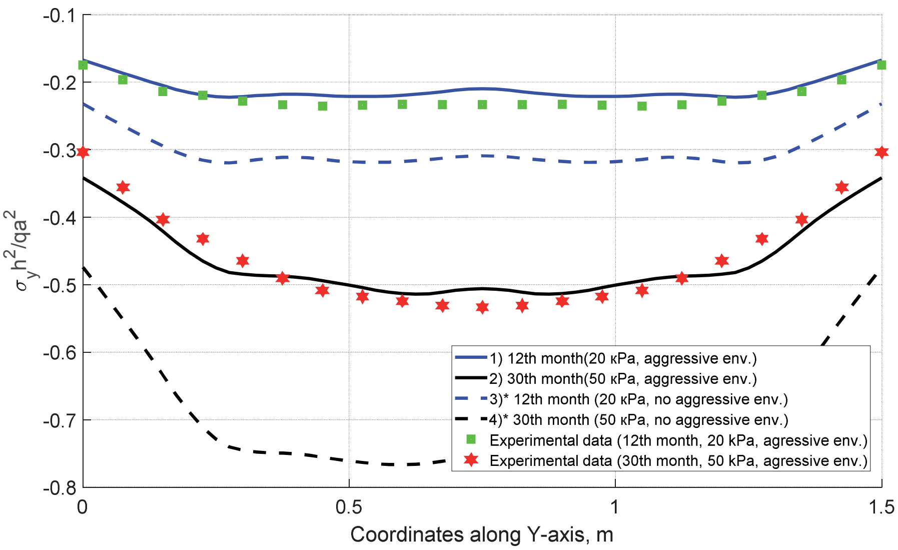


FIGURE 4. Stresses σ_y on the top surface of the concrete slab along OY axis (the axis directed along the short side of the slab)

and an increase in the period of exposure to the aggressive operating environment it reaches 35%. The growth of maximum stresses in reinforced concrete is up to 20% with a period of influence of the aggressive environment up to 12 months, with a period of 30 months or more, the stress increase reaches 56%. Convergence with experimental data (Petrov et al., 1996, 2018; Seljaev, 2006) stays in 5% range for deflections and 10% for stresses.

The results of the work show that as the layer of protective polymer concrete layer damaged, the cracking process is accelerated and the number of cracks along the thickness of the plates increased. Thus, it is proved that in determining the stress--strain state of laminated reinforced plates, it is necessary to consider the multi-modulus behavior of their materials and the impact of an aggressive operating environment.

Conclusions

The analysis of known determinant ratios for materials with complicated properties, exploited in aggressive environments, have a number of disadvantages, not taking into account the important peculiarities of their deformation, which, in many cases, introduces certain model limitations on the characteristics of materials or leads to significant errors of the obtained approximations of experimental data. The solution of applied problems of non-linear mechanics of materials with complicated properties requires the application of sufficiently versatile and reliable determining ratios, as well as improvement of known models of solving specific problems.

The results of the calculation confirm the fact that the accounting of non-linearity of diagrams of deformation and kinetics of aggressive operational environments makes significant adjustments in the stress-strain state of structures, which is especially important when design and verification calculations of building constructions of industrial objects and road network. In particular, under the influence of the aggressive environment, there is an increase in the deflections of the plate working without cracks by 17%, and in the process of cracking with the continuation of this effect – by 35%. There is also a redistribution of stresses in the structure, leading to an increase in the maximum stresses in reinforced concrete by 20% for the time interval of exposure to aggressive environment up to 12 months, and the period up to 30 months or more, this growth reaches 56%. The error of the calculations in comparison with the experiments (Petrov et al. 1996, 2018; Seljaev, 2006) varies in the range of 5% for deflections and 10% for stresses.

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Summary

Determination of stress-strain state of reinforced slabs from non-linear material taking into account the influence of aggressive environment. In this article, the construction of finite-elemental model of definition of stress-strain state of reinforced concrete plates in conditions of active deformation and simple loading in combination with long-term influence of chloride-containing operating environment. Non-linear behavior of concrete is simulated based on the determining relations proposed by Treschev, cracking and plastic deformations in armature are taken into account. The impact of the aggressive environment is taken into account in accordance with the model proposed by Petrov and Penina. In the article all basic correlations of finite elements method in convenient for software realization on a computer are given.

As the object of research for this article is a concrete plate reinforced with steel reinforcement in a stretched area, which is under the joint influence of mechanical load and aggressive chloride-containing environment on the protective polymer–concrete layer. The load was taken evenly distributed across the entire slab area. At the solution of this problem the non--linear sensitivity of the basic material (concrete) to the type of the tense condition, plastic deformations in armature, degradation of a protective concrete at influence of external aggressive environment are taken into account. In the article some especially characteristic results of mathematical modeling of the specified model problem are given. The obtained results of joint influence on the plate of mechanical load and aggressive environment are analyzed.

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